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INCORPORATING FLEXIBLE FABRICATION RATE AND RANDOM SCRAP INTO A FPR-BASED SUPPLY-CHAIN SYSTEM

Abstract. Operating in highly competitive world markets, managers of the present-day manufacturing firms always look for potential means to reduce fabrication cycle length, assure product quality, and trim down shipping expenses in their intra-supply chains. To help accomplish these operation goals, this paper examines a finite production rate (FPR)-based supply-chain system with flexible fabrication rate, random scrap, and multi-ship policy. In which, the manufacture rate in the proposed system is assumes to be adjustable, scrap items are identified and discarded, and fixed-quantity multi-shipment of the finished lot are delivered at a fixed interval of time during production downtime of a cycle. The objective is to determine the optimal lot size and number of shipments that minimize the expected total system costs. Mathematical modeling and Hessian matrix equations are applied to resolve the problem. Applicability of the results is verified via a numerical example. Various desirable system performances are revealed and become accessible to support managerial decision makings.

Key words: Operations Research, Production management, Supply chain, Finite production rate, Flexible fabrication rate, Scrap, Multiple shipments

1. Introduction

A finite production rate (FPR)-based supply chain system with flexible fabrication ate, random scrap, and multi-shipment is investigated in this paper. The classic FPR model [1] considers constant manufacture rate, perfect product quality, and continuous stock issuing policy; and it utilized mathematical modeling and

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differential calculus to decide the most economic batch size in a replenishment cycle. However, in present-day's manufacturing firms, with the intention of reducing fabrication cycle length or leveling machine loading, increase of manufacture rate on equipment can be one of the practical operating options. Buzacott and Ozkarahan [2] studied a two-product single-machine and a two-product two-machine production scheduling problem, wherein the demand is constant and setup cost exists on the two-product single-machine option. It was also assumed that for each product the manufacture rate is adjustable. Their research results revealed that (1) the characteristic of the optimal schedule is not to have any idle time in between runs; (2) if calculation of holding cost is based on average inventory, then on the bottleneck stage maximal manufacture rate always applies to one product; conversely, if maximal inventory constraint exists, then in the single-machine neither product should be fabricated at maximal rate. Gallego [3] examined an economic lot scheduling problem with reduction of the manufacture rate from its standard rate, to stay away from a fast growth of inventories and minimize a sharp lower bound on average stock holding cost. A time-varying heuristic was employed to obtain a near –optimal schedule that helps to decide the item's manufacture rate, order intervals, and lower bound of average holding cost. The research result also indicated that extra savings can be gained if rates can be reduced by inserting random idle times between fabricating successive units of the same item without extra setup costs involved. Eynan [4] investigated a multi-product single machine economic lot scheduling problem, wherein different manufacture rates are permitted in a production run. The optimal common cycle solution to the problem was derived, and lower bounds on savings as compared to conventional approach were also explored and revealed. Glock [5] studied the effect of a variable fabrication rate on maximal stock level and total system costs in a single-product single facility manufacturing system with inventory issuing at discrete time intervals. Several models for the cases of equal- and unequal-sized batch shipments were constructed and investigated. A numerical example was provided to demonstrate that deviating from the standard manufacture rate may reduce stock holding cost and hence lead to savings in total system cost. Farzam Rad et al. [6] considered a multi-product multi-period production planning problem, wherein demand rate, process time, and setup time of every product in each period are of probabilistic nature. Simulation approach was used to help decide the system response rate for various numbers of manufacture rates in different periods, and the optimal response was obtained as well by simulated annealing algorithm. Extra papers that explored various aspects of manufacturing systems with diverse production rates can also be referred to [7-9].

Different from the assumption of perfect product quality in classic FPR model, most real production systems randomly produce scrap items due to a variety of unforeseen issues. Schwaller [10] considered an economic order quantity (EOQ) model incorporating fixed and variable inspection costs in order to find and remove defective products in incoming lots. Cheung and Hausman [11] jointly decided the safety stock and preventive maintenance policies for a manufacturing system that

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is subject to stochastic machine breakdown. Trade-off between investing in these policies were studied and discussed. Both deterministic and exponential repair time distributions were assumed and investigated and the optimal conditions under either one or both policies are derived accordingly. Ritvirool and Ferrell [12] explored the effect of quality issues on a single-vendor, single-buyer procurement system, wherein (Q, r) policy is used by the buyer and fabrication batch size based on make-to-order policy is utilized by the producer. Existence of defective items along with cost of quality is assumed in the producer side. The effect of quality issues on the optimal order quantity, reorder point, and safety stock level were explored. Additional papers that studied different features of manufacturing systems with defective rates and their consequently corrective actions can be referred to [13-16].

In modern transnational firms, joint determination of the optimal fabrication and delivery policies is one of the operating goals that help minimize the overall operation costs. Kohli and Park [17] studied the joint ordering policies with the purpose of reducing transactions cost for multiple products that sold by single seller to multiple customers. In their study, the efficient lot sizes were found to be independent of prices, and were supported by a range of average-unit prices that allow savings on transactions costs between buyer and seller in all locations. In addition, they found out that the product bundling is in favor of efficient joint orders across products. Sarker and Khan [18] determined the optimal lot size for a manufacturing system with a periodic shipping plan. In their proposed system, an ordering policy for raw materials procurement was proposed to meet the requirements of a production facility. Raw materials in lot was processed and transformed into finished products, then transported to meet product demand of buyers at fixed interval of time. A cost function for the proposed manufacturing-delivery integrated system was built and analyzed. A solution procedure was used to help decide the optimal ordering policy for procurement of raw materials and optimal fabrication lot size that to minimize the total system costs. Giri and Chakraborty [19] developed a single-vendor single-buyer coordinated supply chain system, wherein the products in lot were fabricated by the vendor and distributed to buyer in equal shipments. They assumed that the fabrication process may randomly shift from an in-control state to an out-of-control state and generates defective items. The screening process was implemented by the buyer upon receipt of each replenishment lot, and it was further assumed the buyer's inventory deteriorated at a constant rate over time. By minimizing the average cost of proposed system, the optimal vendor-buyer coordinated policy is decided, and through numerical example, the significant cost savings as compared to that in non-coordinated system was obtained. Other papers that addressed a variety of supply-chain systems with multiple deliveries can also be referred to [20-23]. In brief, with the intention of helping production managers to reduce fabrication cycle time, assure product quality, and cut down transportation expenses, this study explores a FPR-based

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supply-chain system with flexible fabrication rate, random scrap, and multi-shipment policy. Since little attention was paid to this field, the present study aims at filling the gap.

2. The proposed FPR-based supply-chain system

A FPR-based supply-chain system with adjustable fabrication, random scrap, and multiple shipments is explored. The parameters used and their definitions are presented in Nomenclature (see Appendix A). Consider that an adjustable rate P_A units per year is adopted by the proposed system to satisfy a demand rate λ units for a particular product. Because of this adjusted higher fabrication rate, manufacturing unit cost C_A and setup cost K_A also increase. The following are the relationships between P_A, C_A, and K_A and their corresponding standard parameters:

$$P_A = (1 + \alpha_1) P \tag{1}$$

$$C_A = (1 + \alpha_3) C \tag{2}$$

$$K_A = (1 + \alpha_2) K \tag{3}$$

where α_1 denotes the adjusted percentage in annual fabrication rate, α_2 and α_3 represent the increase percentages in cost, and P, C, and K are the standard (unadjusted) annual fabrication rate, unit and setup costs, respectively. During uptime t_{1A} of a production cycle (see Figure 1), an x percentage of scrap products is assumed to be randomly produced at a rate d_A as follows:

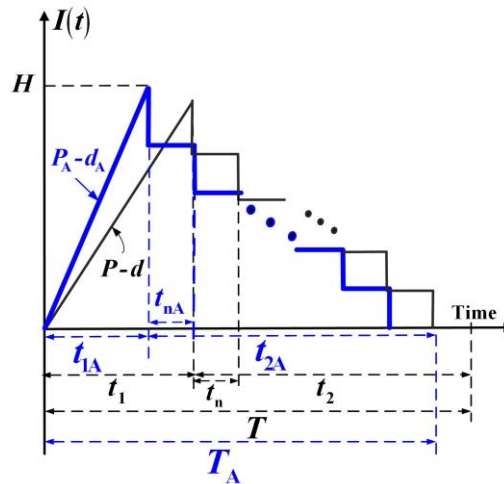$$d_A = xP_A = x(1 + \alpha_1) P \tag{4}$$

Figure 1. Status of inventories in the proposed FPR-based supply-chain system (in blue) compared to that in traditional FPR model (in black)

Total scrap products in the end of t_{1A} are $d_A t_{1A}$ or xQ (see Figure 2), they are discarded when uptime t_{1A} finishes, at unit disposal cost C_S. It is also assumed that

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no shortages are allowed, thus, $(P_A - d_A - \lambda)$ must be greater than zero.
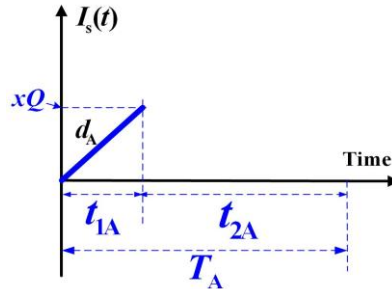

Figure 2. Status of scrap items in proposed FPR-based supply-chain system

By observing Figure 1, the production cycle time T_A, uptime t_{1A}, distribution time t_{2A}, batch size Q, and maximal level of finished products H in a cycle, can be observed as follows:

$$T_A = t_{1A} + t_{2A} = \frac{Q(1-x)}{\lambda} \tag{5}$$

$$t_{1A} = \frac{Q}{P_A} = \frac{Q}{(1+\alpha_1)P} \tag{6}$$

$$t_{2A} = T_A - t_{1A} = \frac{Q(1-x)}{\lambda} - \frac{Q}{(1+\alpha_1)P} \tag{7}$$

$$Q = P_A t_{1A} = \left[(1+\alpha_1)P\right]t_{1A} \tag{8}$$

$$H = (P_A - d_A)t_{1A} \tag{9}$$

Upon completion of production process, n fixed quantity installments of finished products are distributed to customer, at a fixed interval of time in t_{2A} (see Figure 3). The interval of time t_{nA} between two consecutive shipments and number of finished products per shipment D, are as follows:

$$t_{nA} = \frac{t_{2A}}{n} \tag{10}$$

$$D = \frac{H}{n} \tag{11}$$

In downtime t_{2A}, total inventories (Figure 3) are as follows [24]:

$$\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)Ht_{2A} = \left(\frac{n-1}{2n}\right)Ht_{2A} \tag{12}$$

Status of stocks at customer's side in any given cycle is illustrated in Figure 4. In the end of each time interval t_{nA} (after demand λt_{nA} is met) the leftover stocks I is as follows:
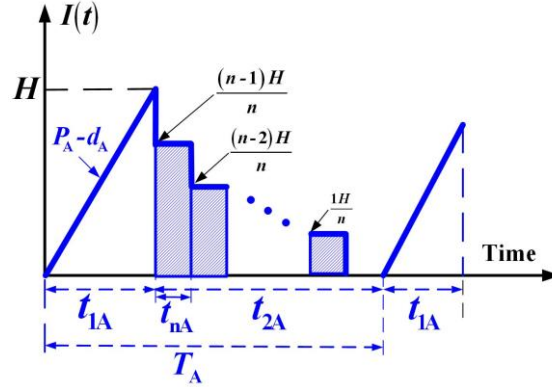
$$I = D - \lambda t_{nA} \tag{13}$$

Figure 3. Status of finished products during t_{2A} in proposed system
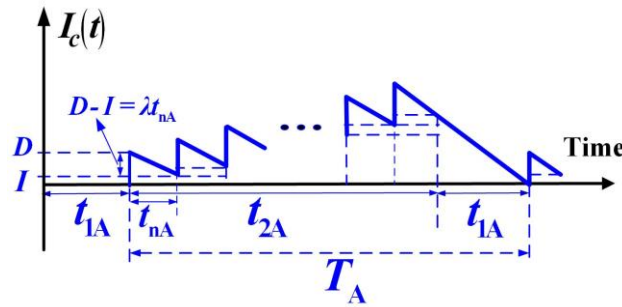

Figure 4. Status of stocks at customer's side in a given cycle in proposed system

In any production cycle, one can obtain total inventories at customer's side (Fig. 4) are as follows:

$$\frac{n(n-1)}{2} I t_{nA} + n\left(D - \frac{\lambda t_{nA}}{2}\right) t_{nA} + \frac{nI}{2}\left(t_{1A}\right) = \frac{1}{2}\left[\frac{H t_{2A}}{n} + T_A\left(H - \lambda t_{2A}\right)\right] \quad (14)$$

Overall cost per cycle of the proposed FPR-based supply-chain system includes the setup cost, variable fabrication and disposal costs, fixed and variable distribution costs, holding costs of finished and scrap products in uptime t_{1A}, holding cost for products in distribution time t_{2A}, and stock holding cost at customer's side in the cycle. Therefore, $TC(Q, n)$ is

$$TC(Q, n) = K_A + C_A Q + C_S\left(xQ\right) + nK_1 + C_T Q + h\left[\frac{H + d_A t_{1A}}{2}\left(t_{1A}\right)\right]$$
$$+ h\left(\frac{n-1}{2n}\right) H t_{2A} + \frac{h_2}{2}\left[\frac{H t_{2A}}{n} + T_A\left(H - \lambda t_{2A}\right)\right] \quad (15)$$

Substitute K_A, C_A, P_A, and d_A in Eq. (15), $TC(Q, n)$ becomes

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$$TC(Q,\, n) = (1+\alpha_2)K + (1+\alpha_3)CQ + C_S xQ + nK_1 + C_T Q + h\left(\frac{n-1}{2n}\right)Ht_{2\mathrm{A}}$$

$$+ h\left[\frac{H + x\left[(1+\alpha_1)P\right]t_{1\mathrm{A}}}{2}\left(t_{1\mathrm{A}}\right)\right] + \frac{h_2}{2}\left[\frac{Ht_{2\mathrm{A}}}{n} + T_\mathrm{A}\left(H - \lambda t_{2\mathrm{A}}\right)\right]$$

(16)

Further replace H, $t_{1\mathrm{A}}$, $t_{2\mathrm{A}}$, and T_A with the right-hand side results from Eqs. (5) to (7) and (9) in Eq. (16), and use the expected values of x to take its randomness into account, and with extra derivations, $E[TCU(Q, n)]$ turns into the following:

$$E\left[TCU(Q, n)\right] = \frac{E\left[TC(Q, n)\right]}{E\left[T_\mathrm{A}\right]} = \lambda\left[(1+\alpha_3)CE_0 + C_S E_1\right] + \frac{\left[(1+\alpha_2)K\right]\lambda E_0}{Q} + \frac{\lambda nK_1 E_0}{Q} + C_T\lambda$$

$$+ \frac{hQ\left(1-E[x]\right)}{2} + \frac{h\lambda QE_1}{2(1+\alpha_1)P} + \frac{Q(h_2-h)}{2n}\left[\left(1-E[x]\right) - \frac{\lambda}{(1+\alpha_1)P}\right] + \frac{h_2 Q}{2}\left[\frac{\lambda}{(1+\alpha_1)P}\right]$$

(17)

where $\qquad E_0 = \dfrac{1}{1-E[x]};\ E_1 = \dfrac{E[x]}{1-E[x]}$

3. Deriving the optimal lot size and number of deliveries per cycle

Apply the Hessian matrix equations [25], and we find the following (refer to Appendix B for details):

$$\begin{bmatrix} Q & n \end{bmatrix} \cdot \begin{pmatrix} \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2} & \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n} \\ \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n} & \dfrac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} = \frac{2\left[(1+\alpha_2)K\right]\lambda E_0}{Q} > 0$$

(18)

Since α_2, K, λ, E_0, and Q are all positive, so Eq. (18) results positive. Therefore, $E[TCU(Q, n)]$ is convex for all Q and n different from zero. To find the optimal Q^* and n^*, we set both first partial derivatives of $E[TCU(Q, n)]$ with respect to Q and n equal to 0, and solve the linear system as follows:

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial Q} = -\frac{\left[(1+\alpha_2)K\right]\lambda E_0}{Q^2} - \frac{nK_1\lambda E_0}{Q^2} + \frac{h(1-E[x])}{2} + \frac{h\lambda E_1}{2(1+\alpha_1)P}$$

$$+ (h_2-h)\left(\frac{1}{2n}\right)\left[(1-E[x]) - \frac{\lambda}{(1+\alpha_1)P}\right] + \frac{h_2\lambda}{2(1+\alpha_1)P} = 0$$

(19)

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial n} = \frac{\lambda K_1 E_0}{Q} + \frac{Q(h-h_2)}{2n^2}\left[(1-E[x]) - \frac{\lambda}{(1+\alpha_1)P}\right] = 0$$

(20)

With further derivatives, optimal production lot size and number of deliveries per cycle can be found as follows:

$$Q^* = \sqrt{\frac{2\lambda\left[(1+\alpha_2)K + nK_1\right]}{h(1-E[x])^2 + \frac{h\lambda E[x]}{(1+\alpha_1)P} + \frac{h_2\lambda(1-E[x])}{(1+\alpha_1)P} + \frac{(h_2-h)\omega}{n}}} \qquad (21)$$

and

$$n^* = \sqrt{\frac{(h_2-h)K(1+\alpha)\omega}{K_1\left[h(1-E[x])^2 + \frac{h\lambda E[x]}{(1+\alpha_1)P} + \frac{h_2\lambda(1-E[x])}{(1+\alpha_1)P}\right]}} \qquad (22)$$

where

$$\omega = \left[(1-E[x])^2 - \frac{\lambda(1-E[x])}{(1+\alpha_1)P}\right] \qquad (23)$$

3.1. Verifications of the obtained results

Let α_1 and α_2 equal to zeros, the proposed FPR-based supply-chain system is converted into a (unadjusted production rate) FPR model with random scraps and multiple deliveries, Eqs. (21) to (23) turn into the following:

$$Q^* = \sqrt{\frac{2\lambda(K + nK_1)}{h(1-E[x])^2 + \frac{h\lambda E[x]}{P} + \frac{h_2\lambda(1-E[x])}{P} + \frac{(h_2-h)\omega}{n}}} \qquad (24)$$

and

$$n^* = \sqrt{\frac{(h_2-h)K\omega}{K_1\left[h(1-E[x])^2 + \frac{h\lambda E[x]}{P} + \frac{h_2\lambda(1-E[x])}{P}\right]}} \qquad (25)$$

where

$$\omega = \left[(1-E[x])^2 - \frac{\lambda(1-E[x])}{P}\right] \qquad (26)$$

Moreover, if no scrap items are produced (i.e., $x = 0$), then the proposed FPR-based supply-chain system is converted into a traditional FPR model as follows:

$$Q^* = \sqrt{\frac{2\lambda(K + nK_1)}{h + \frac{h_2\lambda}{P} + \frac{(h_2-h)\omega}{n}}} \qquad (27)$$

and

$$n^* = \sqrt{\frac{(h_2-h)K\omega}{K_1\left(h + \frac{h_2\lambda}{P}\right)}} \qquad (28)$$

where

$$\omega = \left(1 - \frac{\lambda}{P}\right) \tag{29}$$

4. Numerical example

Applicability of the proposed FPR-based supply-chain system with flexible manufacture rate, random scrap, and multiple shipments is demonstrated using the following numerical example, in which

P = 20,000 products per year,
λ = 4,000 products per year,
α_1 = 0.5,
P_A = 30,000 products per year (i.e., $(1 + \alpha_1)P$),
x = [0, 0.2], which obeys a uniform distribution,
K = \$5,000 per cycle,
C = \$100 per product,
α_2 = 0.1 (i.e., $0.2(\alpha_1)$),
K_A = \$5,500 (i.e., $(1 + \alpha_2)K$),
K = \$5,000 per cycle,
α_3 = 0.25 (i.e., $0.5(\alpha_1)$),
C_A = \$125 (i.e., $(1 + \alpha_3)C$),
h = \$30,
C_S = \$20,
K_1 = \$800 per shipment,
C_T = \$0.5 per product.

Apply Eqs. (21) and (22), we obtain $n^* = 3$ and $Q^* = 1,175$. From computation of Eq. (17), $E[TCU(Q^*, n^*)] = \$626,223$ is found. Further analysis reveals the behavior of different cost components of $E[TCU(Q, n)]$ with respect to Q, as depicted in Figure 5.
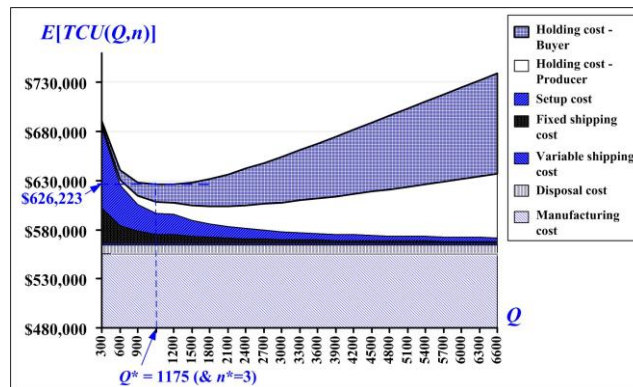

Figure 5. Behavior of different cost components of E[*TCU*(Q, n)] with respect to Q

It is noted that as Q increases, inventory holding costs at both buyer and producer sides significantly grow; on the contrary, both setup cost and fixed shipping cost

notably decrease when Q increases. Effect of different scrap rates on total scrap and disposal costs is displayed in Figure 6. It can be seen from Figure 6 that as random scrap rate x increases, total scrap and disposal costs of the proposed model go up significantly.
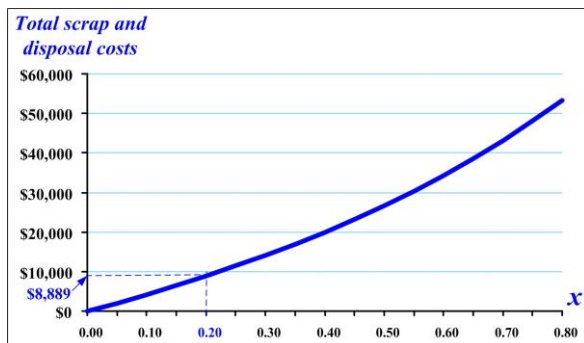

Figure 6.   Effect of different scrap rates on total scrap and disposal costs

Figure 7 reveals the joint impact of variations in production lot size Q and number of deliveries per cycle n on the expected total system cost per unit time $E[TCU(Q, n)]$. It is noted that as both Q and n deviate from Q^* and n^*, the expected total system cost $E[TCU(Q, n)]$ increases accordingly.
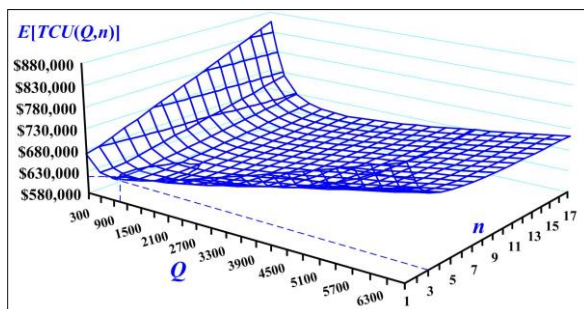

Figure 7.   Joint impact of lot size Q and number of deliveries n on $E[TCU(Q, n)]$

Further analysis (see Figure 8) shows that the joint effects of variations in random scrap rate x and production rate adjusted factor α_1 on $E[TCU(Q, n)]$.
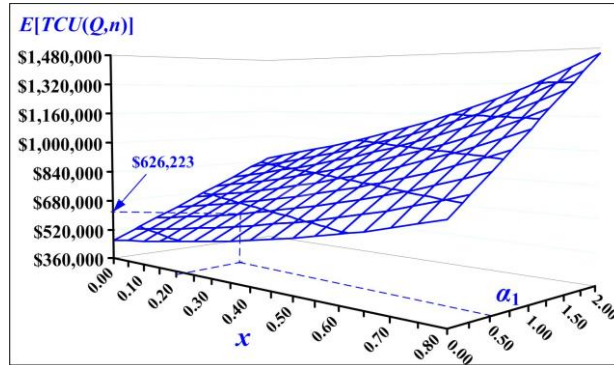
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Figure 8.   **Joint effects of *x* and production rate adjusted factor *α*₁ on *E[TCU(Q, n)]***

It can be seen that as *x* increases, $E[TCU(Q, n)]$ grows accordingly; and as
production rate adjusted factor α_1 increases, $E[TCU(Q, n)]$ goes up significantly,
mainly due to increases in setup and variable production costs. Further analysis
reveals the impact of production rate adjusted factor α_1 on the optimal lot size and
number of deliveries (see Table 1).

Table 1.   **Impact of production rate adjusted factor *α*₁ on the optimal *Q** and *n****

α_1	Q^*	n^*	$E[TCU(Q^*, n^*)]$
0	979	2	$515,237
0.1	995	2	$537,386
0.2	1010	2	$559,608
0.3	1144	2	$581,805
0.4	1160	2	$603,991
0.5	1175	3	$626,223
0.6	1189	3	$648,494
0.7	1202	3	$670,795
0.8	1215	3	$693,122
0.9	1227	3	$715,470
1.0	1239	3	$737,836
1.1	1250	3	$760,218
1.2	1261	3	$782,613
1.3	1272	3	$805,020
1.4	1283	3	$827,435
1.5	1293	3	$849,860
1.6	1303	3	$872,291
1.7	1313	3	$894,729
1.8	1322	3	$917,173
1.9	1332	3	$939,621
2.0	1341	3	$962,073

From Table 1, we can obtain the influence of α_1 on optimal lot size Q and number
of deliveries n as depicted in Figure 9. It can be seen from Figure 9 that as

production rate adjusted factor α_1 increases, both Q and n grow accordingly, and the optimal (Q^*, n^*) policy is at (1175, 3), as α_1 set at 0.50 in our numerical example.
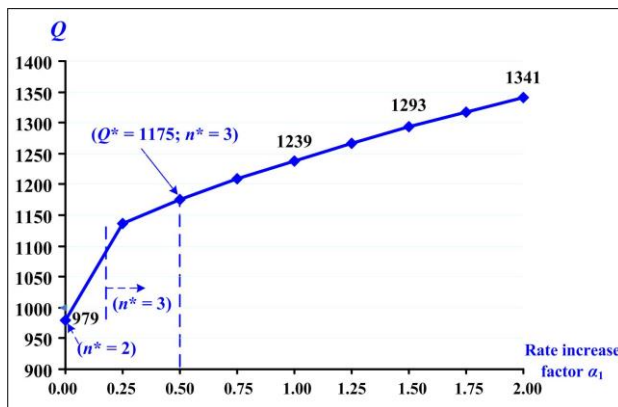

Figure 9.   Influence of production rate adjusted factor α_1 on the optimal Q and n

Figure 10 depicts the effect of production rate adjusted factor α_1 on the machine utilization percentage. It is noted that as production rate adjusted factor α_1 increases, machine utilization percentage declines considerably, and machine utilization percentage is at 14.8% as α_1 set at 0.50 in our numerical example (i.e., utilization reduces 33.3%, from 22.2% to 14.8%, as production rate adjusts 50% higher).
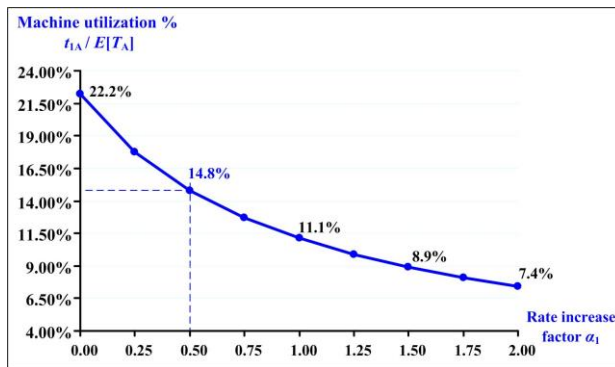
Figure 10. Effect of production rate adjusted factor α_1 on machine utilization percentage

Moreover, the proposed model enables us to expose the impact of unit production cost increase factor α_3 on the expected system cost $E[TCU(Q, n)]$ (see Table 2).

Table 2: **Impact of unit cost increase factor α_3 on $E[TCU(Q, n)]$ and its increases %**

α_3	$E[TCU(Q, n)]$	increase %
0	$515,237	-
0.05	$537,386	4.30%
0.10	$559,608	8.61%

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0.15	$581,805	12.92%
0.20	$603,991	17.23%
0.25	$626,223	21.54%
0.30	$648,494	25.86%
0.35	$670,795	30.19%
0.40	$693,122	34.52%
0.45	$715,470	38.86%
0.50	$737,836	43.20%
0.55	$760,218	47.55%
0.60	$782,613	51.89%
0.65	$805,020	56.24%
0.70	$827,435	60.59%
0.75	$849,860	64.95%
0.80	$872,291	69.30%
0.85	$894,729	73.65%
0.90	$917,173	78.01%
0.95	$939,621	82.37%
1.00	$962,073	86.72%

From Table 2, we can obtain the behavior of $E[TCU(Q, n)]$ with respect to unit production cost increase factor α_3 (as shown in Figure 11). It can be seen that as α_3 increases, $E[TCU(Q, n)]$ goes up notably. In our numerical example, as α_3 set at 0.25, the expected system cost $E[TCU(Q, n)]$ grows 21.54%.
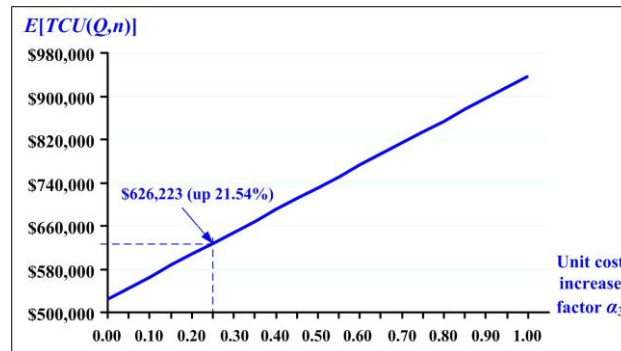
Figure 11.   Behavior of $E[TCU(Q, n)]$ with respect to unit cost increase factor α_3

5. Conclusions

A specific FPR-based supply-chain system is built and explored with the purpose of helping management of the manufacturing firms to reduce fabrication cycle length, assure product quality, and cut down product shipping expenses. Optimization techniques are employed to resolve the problem. The optimal lot size and number of shipments per cycle are determined. Applicability of the research results are verified via a numerical example.

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Sensitivity analyses of the proposed system enable production managers to reveal different critical system information for supporting their decision makings. Such information includes (1) effect of lot-size and scrap rate on expected system cost (Figs. 5-6); (2) joint influence of lot-size and number of shipments on expected system cost (Fig. 7); (3) joint effects of scrap rate and production rate adjusted factor α_1 on expected system cost (Fig. 8); (4) impact of production rate adjusted factor α_1 on optimal lot-size and number of shipments (Fig. 9 & Table 1); (5) effect of production rate adjusted factor α_1 on machine utilization (Fig. 10); (6) the behavior of expected system cost with respect to unit cost increase factor α_3 (Table 2 & Fig. 11); etc. Furthermore, the proposed study can also incorporate the likely machine maintenance cost increase as well as the potential benefits gained due to expedited/adjusted fabrication rate (see the conceptual effect of utilization decrease on the benefit/cost of the proposed system as shown in Figure 12).

In summary, without an in-depth investigation (like the proposed study has done), none of the aforementioned critical system information of this specific FPR-based supply-chain system is accessible to production managers. For future study, examining the effect of reworking nonconforming items on the optimal operating policy to the same problem could be an interesting subject.
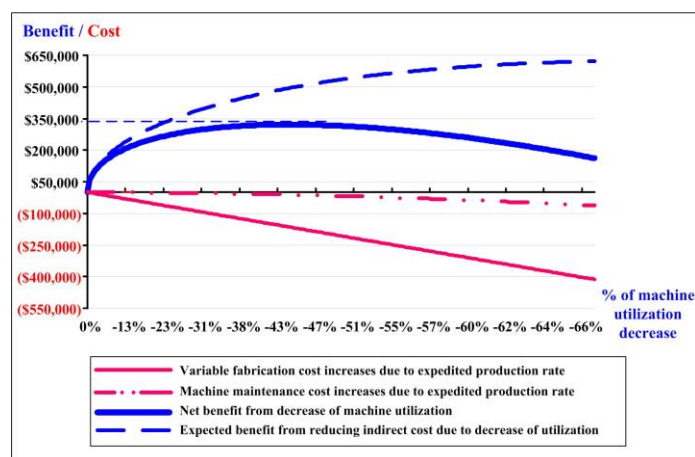

Figure 12.   The effect of utilization decrease on benefit/cost of the proposed system

ACKNOWLEDGMENTS

Incorporating Flexible Fabrication Rate and Random Scrap Into A FPR-Based Supply-Chain System
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Appendix A - Nomenclature

Notation	Definition
λ	Annual demand rate of the proposed FPR-based supply chain system
Q	Lot size per cycle of the proposed system – one of the decision variables
n	Number of deliveries per cycle – the other decision variable
T_A	Cycle time of the proposed system
t_{1A}	Uptime
t_{2A}	Downtime (or distribution time)
t_{nA}	Interval of time between two consecutive deliveries in t_{2A}
P_A	Adjusted production rate of the proposed system
P	Standard production rate in the conventional FPR model
α_1	Adjusted percentage of production rate
K_A	Adjusted setup cost based on the adjusted production rate in the proposed system
K	Standard setup cost
α_2	Adjusted proportion of setup cost in the proposed system
C_A	Adjusted unit production cost in the proposed system
C	Standard unit production cost
α_3	Adjusted proportion of unit production cost in the proposed system
x	Random scrap rate during production process in the proposed system
d_A	Fabrication rate of scrap items in the proposed system
h	Unit holding cost per year
C_S	Unit disposal cost per scrapped item
K_1	Fixed cost per delivery
C_T	Unit distribution cost
D	Number of finished products per delivery
I	the leftover stocks in the end of each t_{nA}, after demand λt_{nA} is satisfied
$E[T_A]$	The expected cycle time of the proposed system
$TC(Q, n)$	Total system cost per cycle of the proposed system
$E[TCU(Q,n)]$	The expected total system cost per unit time in the proposed system
$I(t)$	On-hand inventory level of perfect quality products at time t
$I_d(t)$	On-hand inventory level of scrap items at time t
d	Production rate of scrap items in the traditional model
t_1	Uptime in traditional FPR model
t_2	Downtime in traditional FPR model
t_n	Interval of time between two consecutive deliveries in traditional FPR model
T	Cycle time in traditional FPR model

Appendix B - Hessian matrix equations

Apply the Hessian matrix equations [25], we first obtain the following:

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial Q} = -\frac{\left[(1+\alpha_2)K\right]\lambda E_0}{Q^2} - \frac{nK_1\lambda E_0}{Q^2} + \frac{h(1-E[x])}{2} + \frac{h\lambda E_1}{2(1+\alpha_1)P} \tag{A-1}$$

$$+ \left(h_2 - h\right)\left(\frac{1}{2n}\right)\left[(1-E[x]) - \frac{\lambda}{(1+\alpha_1)P}\right] + \frac{h_2\lambda}{2(1+\alpha_1)P}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q^2} = \frac{2[(1+\alpha_2)K]\lambda E_0}{Q^3} + \frac{2nK_1\lambda E_0}{Q^3} \tag{A-2}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial n} = \frac{\lambda K_1 E_0}{Q} + \frac{Q(h-h_2)}{2n^2}\left[\left(1-E[x]\right) - \frac{\lambda}{(1+\alpha_1)P}\right] \tag{A-3}$$

$$\frac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial n^2} = \frac{-Q(h-h_2)}{n^3}\left[\left(1-E[x]\right) - \frac{\lambda}{(1+\alpha_1)P}\right] \tag{A-4}$$

$$\frac{\partial E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n} = \frac{-\lambda K_1 E_0}{Q^2} + \frac{(h-h_2)}{2n^2}\left[\left(1-E[x]\right) - \frac{\lambda}{(1+\alpha_1)P}\right] \tag{A-5}$$

Substitute Eqs. (A-1) to (A-5) in Hessian matrix equations and with extra derivations, we obtain the following:

$$\begin{bmatrix} Q & n \end{bmatrix} \cdot \begin{pmatrix} \dfrac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q^2} & \dfrac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n} \\ \dfrac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial Q\partial n} & \dfrac{\partial^2 E\left[TCU\left(Q,n\right)\right]}{\partial n^2} \end{pmatrix} \cdot \begin{bmatrix} Q \\ n \end{bmatrix} = \frac{2\left[(1+\alpha_2)K\right]\lambda E_0}{Q} \tag{A-6}$$

REFERENCES

[1] **Taft, E.W. (1918),** *The Most Economical Production Lot. Iron Age 101, 1410-1412;*

[2] **Buzacott, J.A., Ozkarahan, I.A. (1983),** *One- and Two-Stage Scheduling of Two Products with Distributed Inserted Idle Time: The Benefits of a Controllable Production Rate. Naval research logistics quarterly 30(4), 675-696;*

[3] **Gallego, G. (1993),** *Reduced Production Rates in the Economic Lot Scheduling Problem. International Journal of Production Research 31(5), 1035-1046;*

[4] **Eynan, A. (2003),** *The Benefits of Flexible Production Rates in the Economic Lot Scheduling Problem. IIE Transactions 35(11), 1057-1064;*

[5] **Glock, C.H. (2010),** *Batch Sizing with Controllable Production Rates. International Journal of Production Research 48(20), 5925-5942;*

_____

[6]   **Farzam Rad, M., Sajadi, S.M., Hosseinzadeh Kashan, A. (2015),**
        Determination of Optimal Production Rate in Stochastic Manufacturing
        Systems by Simulation Optimization Approach. *International Journal of*
        Industrial and Systems Engineering 20(3), 306-322;

[7]   **Giri, B.C., Dohi, T. (2005),** ***Computational Aspects of an Extended EMQ***
        Model with Variable Production Rate. *Computers and Operations Research*
        32(12), 3143-3161;

[8]   **Sharma, S. (2011),** ***Effects Concerning Quality Level with the Increase in***
        Production Rate. *International Journal of Advanced Manufacturing*
        Technology 53(5-8), 629-634;

[9]   **Sajadieh, M.S., Larsen, C. (2015),** ***A Coordinated Manufacturer-Retailer***
        Model under Stochastic Demand and Production Rate. *International*
        Journal of Production Economics 168, 64-70;

[10]   **Schwaller, R. (1988),** ***EOQ under Inspection Costs.*** *Production and*
        Inventory Management, 29, 22-24;

[11]   **Cheung K. L., Hausman, W. H. (1997),** ***Joint Determination of Preventive***
        Maintenance and Safety Stocks in an Unreliable Production Environment.
        Naval Research Logistics 44(3), 257-272;

[12]   **Ritvirool, A., Ferrell Jr., W. G. (2007),** ***The Effect on Inventory of***
        Cooperation in Single-Vendor, Single-Buyer Systems with Quality
        Considerations. *International Journal of Operational Research 2(3),*
        338-356;

[13]   **Chiu, Y-S.P., Chiang, K-W., Chiu, S.W., Song, M-S. (2016),** ***Simultaneous***
        Determination of Production and Shipment Decisions for a Multi-Product
        Inventory System with a Rework Process. *Advances in Production*
        Engineering & Management 11(2), 141-151;

[14]   **Jindal, P., Solanki, A. (2016),** ***Integrated Supply Chain Inventory Model***
        with Quality Improvement Involving Controllable Lead Time and
        Backorder Price Discount. *International Journal of Industrial Engineering*
        Computations 7(3), 463-480;

[15]   **Makarova, I., Shubenkova, K., Mavrin, V., Boyko, A. (2017),** ***Ways to***
        Increase Sustainibility of the Transportation System. *Journal of Applied*
        Engineering Science 15(1), 89-98;

[16]   **Michalski, G. (2016),** ***Risk Pressure and Inventories Levels. Influence of***
        Risk Sensitivity on Working Capital Levels. *Economic Computation and*
        Economic Cybernetics Studies and Research 50(1), 189-196;

[17]   **Kohli, R., Park, H. (1994),** ***Coordinating Buyer-seller Transactions across***
        Multiple Products. *Management Science 40(9), 45-50;*

[18]   **Sarker, R. A., Khan, L. R. (2001),** ***An Optimal Batch Size under a Periodic***
        Delivery Policy. *International Journal of Systems Science 32(9), 1089-1099;*

[19]   **Giri, B. C., Chakraborty, A. (2011),** ***Supply Chain Coordination for a***
        Deteriorating Product under Stock- Dependent Consumption Rate and
        Unreliable Production Process. *International Journal of Industrial*
        Engineering Computations 2(2), 263-272;

173

[20]   **Cao, J., Chen, Y., Zhou, G. (2015),** *A Novel Statistical Prediction Technique Based on the Dynamic Relationship Identification Algorithm to Forecast Supply Chain Demand.* Economic Computation and Economic Cybernetics Studies and Research 49(1), 203-222;

[21]   **Chiu, Y-S. P., Kuo, J-S., Chiu, S. W., Hsieh, Y-T. (2016),** *Effect of Delayed Differentiation on a Multi–Product Vendor–Buyer Integrated Inventory System with Rework.* Advances in Production Engineering & Management 11(4), 333-344;

[22]   **Florea, A.I., Corbos, R., Popescu, R.I., Zamfir, A. (2016),** *From the Factory Floor to the Shop Floor – Improved Supply Chain for Sustainable Competitive Advantage with Item-Level RFID in Retail.* Economic Computation and Economic Cybernetics Studies and Research 50(4), 119-134;

[23]   **Khalili, A., Ismail, M. Y., Karim, A. N. M., Che Daud, M. R. (2017),** *Critical Success Factors for Soft TQM and Lean Manufacturing Linkage.* Jordan Journal of Mechanical and Industrial Engineering 11(2), 129-140;

[24]   **Chiu, S.W., Liu, C.-J., Li, Y.-Y., Chou, C.-L. (2017),** *Manufacturing Lot Size and Product Distribution Problem with Rework, Outsourcing and Discontinuous Inventory Distribution Policy.* International Journal for Engineering Modelling 30(1-4), 49-61;

[25]   **Rardin, R. L. (1998),** *Optimization in Operations Research.* Prentice-Hall, New Jersey, 739-741.

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